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TO WARD THE THEORY OF DYNAMIC PROBLEMS OF COUPLE-STRESS THERMODIFFUSION OF DEFORMABLE SOLID MICROPOLAR ELASTIC BODIES

Abstract. The matrices of fundamental and other singular solutions are constructed for system of stationary oscillations of the theory of couple-stress thermodiffusion of deformable solid micropolar elastic media in the three-dimensional case and their properties are established. The basic and mixed generalized potentials are constructed and and their boundary and differential properties indicated.

Key words and phrases. Couple-stress thermodiffusion, elasticity, fundamental solutions, potentials.

Connection between processes of strain, torsion-bending, heat conduction and diffusion in micropolar solid isotropic elastic media is described by the second order eight scalar partial differential equations [1]

$$L\left(\frac{\partial}{\partial x},\sigma\right)U(x,\sigma)=0, \tag{1}$$

where $L\left(\frac{\partial}{\partial x},\sigma\right)$ - matrix differential operator of size 8×8

$$L\left(\frac{\partial}{\partial x},\sigma\right)=\left\|L_{ij}\left(\frac{\partial}{\partial x},\sigma\right)\right\|_{8\times 8},$$

$$L_{ij}=\left\|\begin{matrix} L^{(1)} & L^{(2)} \\ L^{(3)} & L^{(4)}\end{matrix}\right\|,\quad i,j=1,...,6,\quad L^{(k)}=\|L^{(k)}_{ij}\|_{3\times 3},\quad k=1,...,4,$$

$$L^{(1)}_{ij}=[(\mu+\alpha)\Delta+\varrho\sigma^2]\delta_{ij}+(\lambda+\mu-\alpha)\frac{\partial^2}{\partial x_i\partial x_j},\quad L^{(2)}_{ij}=L^{(3)}_{ij}=-2\sum_k\varepsilon_{ijk}\frac{\partial}{\partial x_k},$$

$$L^{(4)}_{ij}=[(\nu+\beta)\Delta+(I\sigma^2-4\alpha)]\delta_{ij}+(\varepsilon+\nu-\beta)\frac{\partial^2}{\partial x_i\partial x_j},\quad L_{ij}=i\sigma\gamma_{i-6}\frac{\partial}{\partial x_j},\quad i=7,8,\quad j=1,2,3,$$

$$L_{ij}=0,\quad i=7,8,\quad j=4,5,6,\quad L_{ji}=-\gamma_{i-6}\frac{\partial}{\partial x_j},\quad i=7,8,\quad j=1,2,3,\quad L_{ji}=0,\quad i=7,8,\quad j=4,5,6,$$

$$L_{77}=\delta_1\Delta+i\sigma a_1,\quad L_{88}=\delta_2\Delta+i\sigma a_2,\quad L_{78}=L_{87}=i\sigma a_{12}.$$

$U=(u,\omega,u_7,u_8)^*=\|u_k\|_{8\times 1}$, $u=(u_1,u_2,u_3)^*$ - displacement vector, $\omega=(\omega_1,\omega_2,\omega_3)^*$ - rotation vector, u_7 - temperature variation, u_8 - chemical potential of the domain, $x=(x_1,x_2,x_3)$ - point of Euclidean space E^3, $*$ indicates on the operation of the transformation, Δ - three-dimensional Laplacian operator, δ_{ij} - Kronecker's symbol, ε_{ijk} - Levi-Civita's symbol, $\lambda,\mu,\varrho,I,\nu,\alpha,\beta,\varepsilon,\gamma_1,\gamma_2,\delta_1,\delta_2,a_1,a_2,a_{12}$ - known elastic, thermal and diffusional constants, which satisfy natural restrictions [1,2]

$$\begin{matrix}\varrho>0, & \mu>0, & 3\lambda+2\mu>0, & \alpha>0, & \delta_1>0, & a_1>0, & \gamma_1>0, & \\ I>0, & \nu>0, & 3\varepsilon+2\nu>0, & \beta>0, & \delta_2>0, & a_2>0, & \gamma_2>0, & a_{12}^2-a_1a_2<0.\end{matrix} \tag{3}$$

σ - real or complex parameter.

Let

$$\Phi(x,\sigma)=\|\Phi_{ij}(x,\sigma)\|_{8\times 8}=\|\Phi^1,\Phi^2,...,\Phi^8\|- \tag{4}$$

matrix of fundamental solutions of system (1), where $\Phi^k(x,\sigma)=(\Phi_{1k},\Phi_{2k},...,\Phi_{8k})^*$, $k=1,...,8$ - vector-columns, which satisfy homogeneous equation $L\left(\frac{\partial}{\partial x},\sigma\right)\Phi^k(x,\sigma)=0$, for $\forall x\in E^3\setminus\{0\}$.

Let $\sigma \neq 0$ and we shall seeking $\Phi(x,\sigma)$ in the form

$$\Phi(x,\sigma) = \hat{L}\left(\frac{\partial}{\partial x},\sigma\right)\varphi(x,\sigma), \tag{5}$$

where $\hat{L}\left(\frac{\partial}{\partial x},\sigma\right)$ is connected with $L\left(\frac{\partial}{\partial x},\sigma\right)$ matrix:

$$\hat{L}\left(\frac{\partial}{\partial x},\sigma\right)\cdot L\left(\frac{\partial}{\partial x},\sigma\right) \equiv L\left(\frac{\partial}{\partial x},\sigma\right)\cdot \hat{L}\left(\frac{\partial}{\partial x},\sigma\right) \equiv I detL\left(\frac{\partial}{\partial x},\sigma\right), \tag{6}$$

I - unit matrix of size 8×8. Hence,

$$detL\left(\frac{\partial}{\partial x},\sigma\right)\varphi(x,\sigma) = 0. \tag{7}$$

Direct calculations give

$$detL\left(\frac{\partial}{\partial x},\sigma\right) = \delta_1\delta_2(\lambda+2\mu)(\varepsilon+2\nu)(\mu+\alpha)^2(\nu+\beta)^2(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)\prod_{j=1}^{6}(\Delta+\lambda_j^2), \tag{8}$$

where constants $\lambda_k^2, k=1,...,6$ are defined from the identities

$$\sum_{k=1}^{3}\lambda_k^2 = \frac{\sigma}{(\lambda+2\mu)\delta_1\delta_2}[\varrho\sigma\delta_1\delta_2 + (\lambda+2\mu)i(a_1\delta_2+a_2\delta_1) + i(\delta_1\gamma_2^2+\delta_2\gamma_1^2)],$$

$$\sum_{k=1}^{3}\lambda_{k-1}^2\lambda_k^2 = \frac{\sigma^2}{(\lambda+2\mu)\delta_1\delta_2}[\varrho\sigma i(a_1\delta_2+a_2\delta_1) + (\lambda+2\mu)(a_{12}^2-a_1a_2) + (2\gamma_1\gamma_2a_{12}-a_1\gamma_2^2-a_2\gamma_1^2)], \quad \lambda_0^2\equiv\lambda_3^2, \tag{9}$$

$$\prod_{k=1}^{3}\lambda_k^2 = \frac{\varrho\sigma^4(a_{12}^2-a_1a_2)}{(\lambda+2\mu)\delta_1\delta_2},$$

$$\lambda_4^2+\lambda_5^2 = \frac{\varrho\sigma^2}{\mu+\alpha} + \frac{I\sigma^2-4\alpha}{\nu+\beta} + \frac{4\alpha^2}{(\mu+\alpha)(\nu+\beta)}, \quad \lambda_4^2\cdot\lambda_5^2 = \frac{\varrho\sigma^2}{\mu+\alpha}\cdot\frac{I\sigma^2-4\alpha}{\nu+\beta}, \quad \lambda_6^2 = \frac{I\sigma^2-4\alpha}{\varepsilon+2\nu}.$$

It is shown that λ_k^2, $k=1,...,5$ are complex values.

Noting, that all elements of matrix $\hat{L}\left(\frac{\partial}{\partial x},\sigma\right)$ contain multiplier $(\Delta+_4^2)(\Delta+\lambda_5^2)$: $\hat{L}\left(\frac{\partial}{\partial x},\sigma\right) = \hat{L}_0\left(\frac{\partial}{\partial x},\sigma\right)(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)$, we shall have on account of $\hat{\varphi}(x,\sigma) = \delta_1\delta_2(\lambda+2\mu)(\varepsilon+2\nu)(\mu+\alpha)^2(\nu+\beta)^2\varphi(x,\sigma)$

$$\prod_{k=1}^{6}(\Delta+\lambda_k^2)\hat{\varphi}(x,\sigma) = 0. \tag{10}$$

From this follows

$$\hat{\varphi}(x,\sigma) = \sum_{k=1}^{6} b_k\frac{exp(i\lambda_k|x|)}{|x|}. \tag{11}$$

Picking up constants $b_k, k=1,...,6$ in that way that partial derivatives of $\hat{\varphi}$ of 9-th order will have particlarity $|x|^{-1}$, we shall have (on the assumption of $\lambda_k^2\neq\lambda_j^2, k\neq j=1,...,6$)

$$b_k = \frac{1}{2\pi}\prod_{j=1}^{5}\frac{1}{(\lambda_{k+j}^2-\lambda_k^2)}, \quad k=1,...,6, \lambda_{j+6}=\lambda_j, j=1,...,5.$$

Substituting this founded value of $(\Delta+\lambda_4^2)(\Delta+\lambda_5^2)\varphi = \frac{1}{\delta_1\delta_2(\lambda+2\mu)(\varepsilon+2\nu)(\mu+\alpha)^2(\nu+\beta)^2}\hat{\varphi}$ in (5), we arrive at the seeking matrix of fundamental solutions $\Phi(x,\sigma)$.

Next identity is valid

$$\Phi^*(-x,\sigma) = \tilde{\Phi}(x,\sigma),$$

where $\tilde{\Phi}(x,\sigma)$ - matrix of fundamental solutions of conjugated operator $\tilde{L}\left(\frac{\partial}{\partial x},\sigma\right)$.

Let $D \subset E^3$ - finite area, surrounded by the Liapunov surface S, and U - regular in D vector [2]. From the corresponding Green's formula, in the standard way we obtain representation formula for the regular vector

$$\int_S [\tilde{Q}_{(p)}\Phi^*(x-y,\sigma)]^* P_{(p)} U d_y s - \int_S [\tilde{P}_{(p)}\Phi^*(x-y,\sigma)]^* Q_{(p)} U d_y s -$$

$$\int_D \Phi(x-y,\sigma) L\left(\frac{\partial}{\partial x},\sigma\right) U dy = \begin{cases} 2U(x), \forall x \in D, \\ U(x), \forall x \in S, \\ 0, \forall x \in E^3 \setminus \bar{D}, \end{cases} \qquad (12)$$

$p = 0,1,2,3, \quad \tilde{P}_0 \equiv \tilde{R}, \quad \tilde{Q}_0 \equiv \|\delta_{jk}\|_{8\times 8}$, where

$$RU = \left(HU, \delta_1 \frac{\partial u_7}{\partial n}, \delta_2 \frac{\partial u_8}{\partial n}\right)^*,$$

$$P_{(p)}U = \left(HU, -(\delta_{1p}+\delta_{2p})u_7 + \delta_{3p}\delta_1 \frac{\partial u_7}{\partial n}, -(\delta_{1p}+\delta_{3p})u_8 + \delta_{2p}\delta_2 \frac{\partial u_8}{\partial n}\right)^*, p = 1,2,3,$$

$$Q_{(p)}U = \left(\ddot{U}, (\delta_{1p}+\delta_{2p}\delta_1 \frac{\partial u_7}{\partial n} - \delta_{3p}u_7, (\delta_{1p}+\delta_{3p})\delta_2 \frac{\partial u_8}{\partial n} - \delta_{2p}u_8\right)^*, p = 1,2,3,$$

$HU = (T(u,\omega) - \gamma_1 n u_7 - \gamma_2 n u_8), T$ - stress operator of couple-stress elasticity [2]. $\tilde{R}, \tilde{P}_{(p)}, \tilde{Q}_{(p)}$ - conjugated operators.

Formulas (12) indicate on the construction of basic potentials of the theory of couple-stress thermodiffusion. Investigation of these potentials is passing by the same scheme, which is indicated in [2]. All theorems hold, with the corresponding alterations for the potentials of the couple-stress thermodiffusion.

References

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