Analytical Modeling of the Queuing System in a MATLAB Software Package

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Abstract – The paper dwells on the Pension Fund public services assurance process, as analytical modeling of the queuing system (QS), for the purpose of increasing the effectiveness of its operation; for developing a mathematical model, there has been analyzed the single-channel QS of the pension window. The set problem has been resolved by using a MATLAB software package. As a result of modeling, there have been calculated the system's absolute, nominal and relative carrying capacities. The paper also provides recommendations worked out on the basis of the analysis.

Key words – modeling, public services, carrying capacity, intensity, effectiveness.

I. Introduction

An analytical model is a mathematical description of the system structure and the functioning of the process, as well as the methodology for measuring its performance indicators. Such model allows for describing the system behavior quickly and with high accuracy [1].

An analytical model is especially effective and illustrative when designing the queuing systems (QS). The service requests are migrating to the queuing systems in random time. At this time, the received requests are served through the channels maintained by the system. For posing of a task of providing a complete description and study of QS it is necessary to determine the system structure and service discipline (rule), as well as indicator of the quality of service and the effectiveness, i.e. some numerical rating, by a value of which it would possible to judge the functioning and quality of the studied QS [2]. In the management of business processes, there is a set of problems, which we can consider as QS. As such kind of business process, it is possible to present the process of public services, as QS. The given paper dwells on analytical modeling of the queuing system in the case of the process of public services.

The analytical solutions of queuing problems, which are widely used in applied practice, describe the stationary period of working of system and they are constructed with use of a Poisson stream of events. At the same time, in too time the analysis of duration of transition periods in many cases is characterized by fact that they can constitute an essential part of the working period of system, thus by ignoring the instability period, it is impossible to optimize the system's performance data in general that is very characteristic for the mass services in trade enterprises [2].

II. Basic part

1. *Development of a mathematical model*

Consider the single channel system with refusal, which represents the Pension Fund's window for pension social increments. The request is a citizen, who applies to the Pension Fund. When the window is occupied, his meets with a refusal. The intensity of the flow of requests - y = 1 (one request for 10 minutes). The average service time (service is provided by the State) is 18 minutes. The flow of requests (citizen) and the flow of services (service is provided by the State) are the simplest.

It is required to determine the below listed indicators in the planned regime:

- q - Relative carrying capacity;
- a – Absolute carrying capacity;
- p – Probability of refusals;

- Compare actual carrying capacity of QS with nominal carrying capacity, if the process of providing the State services would require 18 minutes and the citizens would apply to the Pension Fund consistently and continually;
- Find static error (the number of applications of citizens to the Pension Fund, th number of citizens receiving public services, the number refusals to delivery of services during system operation) during 90 minutes of the working period.

Servicing time is determined as a random value distributed according to Poisson's law [3].

Let's introduce the parameter identifiers of QS, which are known from statement of problem:

TFIow = 0.1; % the intensity of the requests of citizens for public services;

TServe = 18; % average servicing time (minutes);

TWork = 90; % QS operating period (minutes).

To resolve the set problem, describe the variables in a MATLAB open file:

```
z = [0 0 0 ]; % system's statements (the number of applicants during the operating period of QS, the number of citizens received public services, the number of citizens remained without public services);
smo_q = 0;  % QS relative carrying capacity;
smo_a = 0;  % QS absolute carrying capacity;
smo_n = 0;  % QS nominal carrying capacity;
p_fail = 0;    % QS probability of refusals;
mu =   1/TServe; % the intensity the flows of services for citizens.
```

Following the announcement of the variables, write the formulas for calculation of the above stated values:

```
smo_q  = mu/(TFIow+mu);
smo_a  =  mu*TFIow/(mu+TFIow);
smo_n  = mu;
p_fail = 1 – smo_q;
```

In order to carry out static modeling, it is necessary to compose the following procedure: the variable z(1) is given a value of the received requests, the variable z(3) is given a number of refusals and the variable z(2) – is a number of the serviced requests. The function *n* serves the modeling of the process of receipt and initialization of the requests, as a random value, which is distributed according to Poisson's law, with a 10-minute mathematical expectation. Calculation of time is carried out in minutes, and it is modeled as a cycle with *t*. On average, one requests is received every 10 minutes, and the event $n = 0$ means that the requests entered the QS. Servicing time is determined by the variable *t*, which is initialized as a random value of a 10-minute mathematical expectation distributed according to Poisson's law. Service completion time for the request is stored in the variable t_{ad}. If t = 0, the service channel is free, and the request entered the QS will be serviced. If t > 0, the request entered the QS is refused that is specified as z(3) = z(3) + 1, but the value of *t* is reduced by one minute in each cycle ($t = t - 1$).

```
for  (y= 1:2)
z = [0  0  0];
t = 0;
n = 0;
for (i= 1:TWork);
if  (n = = 0)  % receipt of the requests;
z(1) = z(1)+1; n = poissrnd(1/Tflow);
if  (t = = 0)  % the requests is received for servicing;
z(2) = z(2) +1;  t = poissrnd(Tserve);
eIse   % the system is occupied, the request is refused
z(3) = z(3)+1;
end
end % the request servicing;
   if  (t > 0)  t = t – 1;
   end
    if  (n > 0)  n = n – 1;
    end
    end.
```

2. *The analytical modeling results*

In order to obtain the modeling results, let's use the GUIDE medium, which is part of a MATLAB package [4, 5], and this latter one allows for developing the interface for the analytical model. The modeling results are presented in Fig. 1.

Thus and so, as a result of analytical modeling of the process of the Pension Fund public services, there has been calculated the relative and absolute carrying capacities of the system, which are equal to 0,47047 and 0,0470, accordingly. Also, there has been calculated the nominal carrying capacity, which is equal to 0,68834.

In addition, as a result of analytical modeling (during 90 minutes of the working period of QS), we determined the static assessment characteristics of QS, which showed that the total number of citizens applied to the Pension Fund (during the period of modeling) is 10 people, including 5 persons serviced immediately.

III. Conclusion

As a result of analytical modeling of the public services assurance process, the main performance characteristics of QS have been determined.

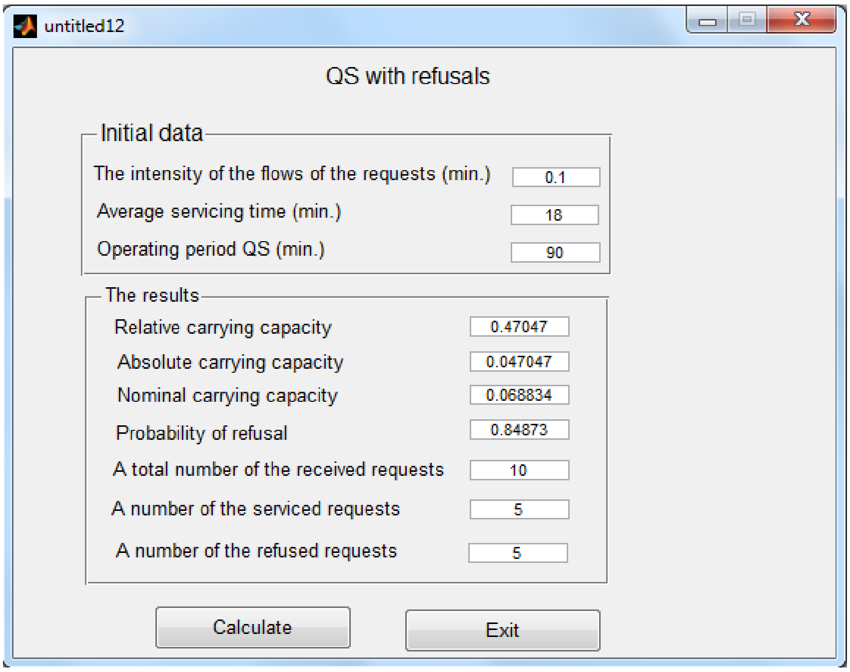

Fig. 1. The results of analytical modeling of QS

When comparing the actual and nominal carrying capacities of QS, it was established that the nominal one (0,068834/0,047047≈ 1,4) is by 1,4-times higher than its actual value, which was calculated by taking into account the random nature of flow of the requests and servicing time. This indicates that the system is not functioning effectively. In order to increase the effectiveness of the Pension Fund, it is necessary to find more optimal parameters of QS (to reduce actual servicing time for citizens or to add one more service channel, i.e. an additional window).

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